REMARKS

The Office Action dated June 15, 2005, has been received and carefully considered. In this response, claims 17-20 have been added. Entry of added claims 17-20 is respectfully requested. Reconsideration of the outstanding objections/rejections in the present application is also respectfully requested based on the following remarks.

At the outset, Applicants note with appreciation the indication on page 6 of the Office Action that claims 6-8 and 14-16 would be allowable if rewritten in independent form including all of the limitations of the base claim and any intervening claims. However, Applicants have opted to defer rewriting the above-identified claims in independent form pending reconsideration of the arguments presented below with respect to the rejected claims.

I. THE OBVIOUSNESS REJECTION OF CLAIMS 1, 3-5, 9, and 11-13

On pages 2-5 of the Office Action, claims 1, 3-5, 9, and 11-13 were rejected under 35 U.S.C. § 103(a) as being unpatentable over Rasala et al. (U.S. Patent No. 6,487,332) in view of Gu et al. (Efficient Protocols for Permutation Routing on All-Optical Multistage Interconnection Networks, pgs. 513-520, Proceedings of the International Conference on Parallel

Processing, August 21-24, 2000). This rejection is hereby respectfully traversed.

As stated in MPEP § 2143, to establish a prima facie case of obviousness, three basic criteria must be met. First, there must be some suggestion or motivation, either in the references themselves or in the knowledge generally available to one of ordinary skill in the art, to modify the reference or to combine reference teachings. Second, there must be a reasonable expectation of success. Finally, the prior art reference (or references when combined) must teach or suggest all the claim limitations. The teaching or suggestion to make the claimed combination and the reasonable expectation of success must both be found in the prior art, not in applicant's disclosure. In re Vaeck, 947 F.2d 488, 20 USPQ2d 1438 (Fed. Cir. 1991). Also, as stated in MPEP § 2143.01, obviousness can only be established by combining or modifying the teachings of the prior art to produce the claimed invention where there is some teaching, suggestion, or motivation to do so found either in the references themselves or in the knowledge generally available to one of ordinary skill in the art. In re Fine, 837 F.2d 1071, 5 USPQ2d 1596 (Fed. Cir. 1988); In re Jones, 958 F.2d 347, 21 USPQ2d 1941 (Fed. Cir. 1992). The mere fact that references can be combined or modified does not render the resultant combination obvious

unless the prior art also suggests the desirability of the combination. In re Mills, 916 F.2d 680, 16 USPQ2d 1430 (Fed. Cir. 1990). Further, as stated in MPEP § 2143.03, to establish *prima facie* obviousness of a claimed invention, all the claim limitations must be taught or suggested by the prior art. In re Royka, 490 F.2d 981, 180 USPQ 580 (CCPA 1974). That is, "[a]ll words in a claim must be considered in judging the patentability of that claim against the prior art." In re Wilson, 424 F.2d 1382, 165 USPQ 494, 496 (CCPA 1970). Additionally, as stated in MPEP § 2141.02, a prior art reference must be considered in its entirety, i.e., as a whole, including portions that would lead away from the claimed invention. W.L. Gore & Associates, Inc. v. Garlock, Inc., 721 F.2d 1540, 220 USPQ 303 (Fed. Cir. 1983), cert. denied, 469 U.S. 851 (1984). Finally, if an independent claim is nonobvious under 35 U.S.C. 103, then any claim depending therefrom is nonobvious. In re Fine, 837 F.2d 1071, 5 USPQ2d 1596 (Fed. Cir. 1988).

Regarding claims 1 and 9, the Examiner asserts that Rasala et al. teaches a method/apparatus for interchanging wavelengths in a multi-wavelength system having W wavelength channels comprising: selectively directing a pair of adjacent frequency channels corresponding to a respective pair of adjacent wavelength channels based upon a routing algorithm (see

wavelength switches 13 in Figure 3; column 2, lines 6-11); interchanging the frequencies of the selectively directed pair of adjacent frequency channels (see wavelength interchangers 11 in Figure 3); and selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels (see fabric 21 in Figure 3). The Examiner acknowledges that Rasala et al. fails to teach selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels based upon a binary representation of each interchanged frequency. However, the Examiner goes on to assert that the present application defines shifting as routing, which is well known in the art.

First of all, nowhere in the present application do Applicants define shifting as routing. Granted, claims 5 and 13 recite that shifting may at least partially include routing. However, selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels based upon a binary representation of each interchanged frequency, as claimed, may involve much more than routing as set forth in the specification and claims.

Secondly, it is respectfully submitted that Rasala et al. fails to even teach selectively shifting the interchanged frequencies of the selectively directed pair of adjacent

frequency channels, as claimed. Specifically, the Examiner asserts that the fabric 21 in Figure 3 of Rasala et al. teaches this claim element. However, the fabric 21 in Figure 3 of Rasala et al. is merely a strictly non-blocking switch fabric for routing signals. In fact, Rasala et al. explicitly discloses that the fabric 21 does not include any devices for changing the wavelength (or hence frequency) of any signal (see column 8, lines 7-9).

Thirdly, it is respectfully submitted that shifting frequencies based upon a binary representation of each frequency is not well known in the art. And despite the Examiner's assertions, it is respectfully submitted that Gu et al. does not teach shifting frequencies based upon a binary representation of each frequency. In contrast, Gu et al. merely discloses labeling inputs (see page 515), and does not provide a binary representation of frequency, let alone shift frequencies based upon a binary representation of each frequency, as claimed.

Furthermore, the Gu et al. reference is a paper that was presented on August 21-24, 2000 (see **full** reference in Appendix B). Thus, the Gu et al. reference has an effective publication date of August 21, 2000.

Applicants respectfully submit that the invention disclosed and claimed in the present application was conceived prior to

August 21, 2000. Applicants also respectfully submit that they were duly diligent in preparing and filing the present application from the date of conception of the invention disclosed and claimed in the present application to the filing date of the present application (i.e., December 29, 2000). Applicants support the above-stated submissions with inventor declarations under 37 C.F.R. § 1.131, which are submitted herewith, and which contain a showing of facts that clearly establish the above-stated submissions. Accordingly, the Gu et al. reference is not a proper prior art reference for application against the claims of the present application.

Accordingly, it is respectfully submitted that claims 1 and 9 would not have been obvious in view of Rasala et al. in view of Gu et al..

Claims 3-5 and 11-13 are dependent upon independent claims 1 and 9, respectively. Thus, since independent claims 1 and 9 should be allowable as discussed above, claims 3-5 and 11-13 should also be allowable at least by virtue of their dependency on independent claims 1 and 9. Moreover, as acknowledged by the Examiner, these claims recite additional features which are not claimed, disclosed, or even suggested by the cited references taken either alone or in combination.

At this point it should be noted that claims 17-20 have been added to clearly recite certain novel features supported by the specification.

In view of the foregoing, it is respectfully requested that the aforementioned obviousness rejection of claims 1, 3-5, 9, and 11-13 be withdrawn.

II. THE OBVIOUSNESS REJECTION OF CLAIMS 2 and 10

On pages 5-6 of the Office Action, claims 2 and 10 were rejected under 35 U.S.C. § 103(a) as being unpatentable over Rasala et al. (U.S. Patent No. 6,487,332) in view of Gu et al. (Efficient Protocols for Permutation Routing on All-Optical Multistage Interconnection Networks, pgs. 513-520, Proceedings of the International Conference on Parallel Processing, August 21-24, 2000) and further in view of Dragone (U.S. Patent Application Publication No. 2003/0091271). This rejection is hereby respectfully traversed.

Regarding claims 2 and 10, these claims are dependent upon independent claims 1 and 9, respectively. Thus, since independent claims 1 and 9 should be allowable as discussed above, claims 2 and 10 should also be allowable at least by virtue of their dependency on independent claims 1 and 9. Moreover, as discussed above, these claims recite additional

features which are not claimed, disclosed, or even suggested by the cited references taken either alone or in combination. For example, the Examiner asserts that Dragone teaches selectively switching the pair of adjacent frequency channels to one of two output pairs, as claimed (see Figure 5B). Applicants respectfully disagree. In contrast, Dragone teaches switching a first signal in a first pair of signals to a first of two output signal pairs while switching a second signal in the first pair of signals to a second of two output signal pairs. This is clearly different than the claimed limitation of switching a pair of adjacent frequency channels to one of two output pairs (i.e., the pair of adjacent frequency channels remain together in either one of two output pairs). Accordingly, it is respectfully submitted that claims 2 and 10 would not have been obvious in view of Rasala et al. in view of Gu et al. and further in view of Dragone.

Additionally, the Dragone reference was filed October 17, 2002, as a divisional patent application of U.S. Patent Application No. 09/629,299, filed July 31, 2000 (now abandoned). Thus, the Dragone reference has an effective filing date of July 31, 2000.

Applicants respectfully submit that the invention disclosed and claimed in the present application was conceived prior to

July 31, 2000. Applicants also respectfully submit that they were duly diligent in preparing and filing the present application from the date of conception of the invention disclosed and claimed in the present application to the filing date of the present application (i.e., December 29, 2000). Applicants support the above-stated submissions with inventor declarations under 37 C.F.R. § 1.131, which are submitted herewith, and which contain a showing of facts that clearly establish the above-stated submissions. Accordingly, the Dragone reference is not a proper prior art reference for application against the claims of the present application.

In view of the foregoing, it is respectfully requested that the aforementioned obviousness rejection of claims 2 and 10 be withdrawn.

III. CONCLUSION

In view of the foregoing, it is respectfully submitted that the present application is in condition for allowance, and an early indication of the same is courteously solicited. The Examiner is respectfully requested to contact the undersigned by telephone at the below listed telephone number, in order to expedite resolution of any issues and to expedite passage of the

present application to issue, if any comments, questions, or suggestions arise in connection with the present application.

To the extent necessary, a petition for an extension of time under 37 CFR § 1.136 is hereby made.

Please charge any shortage in fees due in connection with the filing of this paper, including extension of time fees, to Deposit Account No. 50-0206, and please credit any excess fees to the same deposit account.

Respectfully submitted,

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APPENDIX A

1 (Original). A method for interchanging wavelengths in a multi-wavelength system having W wavelength channels, the method comprising the steps of:

selectively directing a pair of adjacent frequency channels corresponding to a respective pair of adjacent wavelength channels based upon a routing algorithm;

interchanging the frequencies of the selectively directed pair of adjacent frequency channels; and

selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels based upon a binary representation of each interchanged frequency.

2 (Original). The method as defined in claim 1, wherein the step of selectively directing the pair of adjacent frequency channels comprises the step of:

selectively switching the pair of adjacent frequency channels to one of two output pairs.

3 (Original). The method as defined in claim 1, wherein the step of interchanging the frequencies of the selectively directed pair of adjacent frequency channels comprises the step of:

routing the selectively directed pair of adjacent frequency channels based upon a binary representation of the frequency of each of the selectively directed pair of adjacent frequency channels.

4 (Original). The method as defined in claim 3, wherein the step of interchanging the frequencies of the selectively directed pair of adjacent frequency channels further comprises the steps of:

shifting the frequency of a first of the selectively directed pair of adjacent frequency channels by an amount defined by $+\Delta f$; and

shifting the frequency of a second of the selectively directed pair of adjacent frequency channels by an amount defined by $-\Delta f$;

wherein Δf is the frequency spacing between the pair of adjacent frequency channels.

5 (Original). The method as defined in claim 1, wherein the step of selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels comprises the step of:

routing the selectively directed pair of adjacent frequency

channels based upon the binary representation of each interchanged frequency.

6 (Original). The method as defined in claim 5, wherein the step of selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels further comprises the step of:

shifting the frequency of at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $\pm(2^h-1)\Delta f$, wherein $h=0,\ldots,w-1$, $w=\log_2 W$, and Δf is the frequency spacing between the pair of adjacent frequency channels.

7 (Original). The method as defined in claim 5, wherein the step of selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels further comprises the steps of:

shifting the frequency of at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $-2^h\Delta f$;

increasing the shifted frequency of the at least one of the selectively directed pair of adjacent frequency channels; and

shifting the increased shifted frequency of the at least

one of the selectively directed pair of adjacent frequency channels by an amount defined by +Δf;

wherein $h = 0,\ldots,w-1$, $w = \log_2 W$, and Δf is the frequency spacing between the pair of adjacent frequency channels.

8 (Original). The method as defined in claim 5, wherein the step of selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels further comprises the steps of:

shifting the frequency of at least one of the selectively directed pair of adjacent frequency channels by an amount defined by -Δf;

decreasing the shifted frequency of the at least one of the selectively directed pair of adjacent frequency channels; and

shifting the decreased shifted frequency of the at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $+2^h\Delta f$;

wherein $h = 0,\ldots,w-1$, $w = \log_2 W$, and Δf is the frequency spacing between the pair of adjacent frequency channels.

9 (Original). An apparatus for interchanging wavelengths in a multi-wavelength system having W wavelength channels, the

apparatus comprising:

a switching element for selectively directing a pair of adjacent frequency channels corresponding to a respective pair of adjacent wavelength channels based upon a routing algorithm;

a state changer for interchanging the frequencies of the selectively directed pair of adjacent frequency channels; and

a connection module for selectively shifting the interchanged frequencies of the selectively directed pair of adjacent frequency channels based upon a binary representation of each interchanged frequency.

10 (Original). The apparatus as defined in claim 9, the switching element comprises:

a cross-connect for selectively switching the pair of adjacent frequency channels to one of two output pairs.

11 (Original). The apparatus as defined in claim 9, wherein the state changer comprises:

a router for routing the selectively directed pair of adjacent frequency channels based upon a binary representation of the frequency of each of the selectively directed pair of adjacent frequency channels.

12 (Original). The apparatus as defined in claim 11, wherein the state changer further comprises:

a first frequency shifter for shifting the frequency of a first of the selectively directed pair of adjacent frequency channels by an amount defined by +Δf; and

a second frequency shifter for shifting the frequency of a second of the selectively directed pair of adjacent frequency channels by an amount defined by -Δf;

wherein Δf is the frequency spacing between the pair of adjacent frequency channels.

13 (Original). The apparatus as defined in claim 9, wherein the connection module comprises:

a router for routing the selectively directed pair of adjacent frequency channels based upon the binary representation of each interchanged frequency.

14 (Original). The apparatus as defined in claim 13, wherein the connection module further comprises:

at least one frequency shifter for shifting the frequency of at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $\pm(2^h-1)\Delta f$, wherein $h=0,\ldots,w-1$, $w=\log_2 W$, and Δf is the frequency spacing between the

pair of adjacent frequency channels.

15 (Original). The apparatus as defined in claim 13, wherein the connection module further comprises:

a first frequency shifter for shifting the frequency of at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $-2^h\Delta f$;

an increasing up-converter for increasing the shifted frequency of the at least one of the selectively directed pair of adjacent frequency channels; and

a second frequency shifter for shifting the increased shifted frequency of the at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $+\Delta f$;

wherein $h = 0,\ldots,w-1$, $w = \log_2 W$, and Δf is the frequency spacing between the pair of adjacent frequency channels.

16 (Original). The apparatus as defined in claim 13, wherein the connection module further comprises:

a first frequency shifter for shifting the frequency of at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $-\Delta f$;

an increasing down-converter for decreasing the shifted

frequency of the at least one of the selectively directed pair of adjacent frequency channels; and

a second frequency shifter for shifting the decreased shifted frequency of the at least one of the selectively directed pair of adjacent frequency channels by an amount defined by $+2^h\Delta f$;

wherein $h=0,\dots,w-1$, $w=\log_2 W$, and Δf is the frequency spacing between the pair of adjacent frequency channels.

17 (New). A method for interchanging wavelengths in a multi-wavelength system having W wavelength channels, the method comprising the steps of:

selectively switching a pair of adjacent optical frequency signals corresponding to a respective pair of adjacent wavelength channels to one of two output signal pairs;

interchanging the frequencies of the selectively switched pair of adjacent optical frequency signals from one of the two output signal pairs; and

selectively shifting the interchanged frequencies of the selectively switched pair of adjacent optical frequency signals based upon a value of a binary representation of each interchanged frequency.

18 (New). The method as defined in claim 17, further comprising:

selectively shifting the frequencies of the selectively switched pair of adjacent optical frequency signals from another one of the two output signal pairs based upon a value of a binary representation of each interchanged frequency.

19 (New). An apparatus for interchanging wavelengths in a multi-wavelength system having W wavelength channels, the apparatus comprising:

a switching element for selectively switching a pair of adjacent optical frequency signals corresponding to a respective pair of adjacent wavelength channels to one of two output signal pairs;

a state changer for interchanging the frequencies of the selectively switched pair of adjacent optical frequency signals from one of the two output signal pairs; and

a connection module for selectively shifting the interchanged frequencies of the selectively switched pair of adjacent optical frequency signals based upon a value of a binary representation of each interchanged frequency.

20 (New). The apparatus as defined in claim 19, wherein the connection module also selectively shift the frequencies of the

selectively switched pair of adjacent optical frequency signals from another one of the two output signal pairs based upon a value of a binary representation of each interchanged frequency.

APPENDIX B

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Efficient Protocols for Permutation Routing on All-Optical Multistage Interconnection Networks

Qian-Pin Gu* Shietung Peng†

Abstract: *To realize a routing request R on a WDM (wavelength division multiplexing) all-optical network, one needs to set up routing paths in the network for every input-output pair in R and to assign a wavelength to each path so that the paths with the same wavelength are edge-disjoint. The optical bandwidth of the WDM all-optical network is the number of wavelengths supported by each link of the network. If the number of wavelengths for realizing R is at most the bandwidth of the network then R can be routed in one round of routing. However, when the number of wavelengths needed is beyond the bandwidth, multiple rounds of routing are required. In this case, it is important to minimize the number of rounds of routing. We give two deterministic algorithms for permutation routings on the parallel computing systems connected by all-optical multistage interconnection networks (MINs). For the parallel computing systems with practical size, the first algorithm realizes any permutation in two rounds of routing. The second algorithm realizes any bit-permute-complement (BPC) permutation on the n-dimensional MINs in two rounds of routing.*

Key words: Permutation routing, WDM all-optical networks, routing algorithms, multistage interconnection networks, edge-disjoint paths, BPC permutations

1 Introduction

All optical-networks use optical switches to keep the message transmitted in optical from input to output. This approach eliminates the well-known bottleneck of electro-optic conversions at electronic switches and increases greatly the performance of the optical network. Much work has been done on routing problems on all-optical networks [5, 17, 14, 15, 28, 3, 21, 20, 16, 6]. Since the switching is done optically and there is no optical random access memory available, there can be no data storage inside the switches and the circuit-switched routing mode is used for all-optical networks. WDM (wavelength division multiplexing) is one of the most well-known approach to realize the high capacity of all-optical networks. A WDM all-optical network consists of routing nodes (optical switches) and node-to-node links (optical fibers). Each link supports a fixed number of wavelengths. The number of wavelengths supported by each link is the optical bandwidth. We will use this bandwidth, denoted by W, as the bandwidth of the all-optical network. To realize a routing request on the a all-optical network, one needs to set up the routing paths for every input-output pair in the routing request and to assign a wavelength to each path so that the paths which receive the same wavelength are edge-disjoint. It is well understood that the bandwidth of all-optical networks is a scarce resource; according to the previous studies [14, 3], no more than 30-40 in the laboratory, at most half in practical networks, and no anticipation of dramatic progress in the near future. Optimizing the number of wavelengths for routing on WDM all-optical networks has attracted much attention [14, 11, 8, 7, 15, 20, 27, 9, 16, 6]. For a routing request R which needs l wavelengths, if $l \le W$ then R can be realized on the network by one *round of routing*. However, when $l > W$, multiple rounds of routing are needed for realizing R. In this case, it is important to minimize the number of routing rounds for realizing R.

In this paper, we consider this minimization problem for permutation routings on the parallel computing systems connected by all-optical multistage interconnection networks (MINs). The MINs have been used for parallel computing systems such as IBM SP1/SP2 [22] and NEC Cenju-3 [10], and used for the internal structures of optical couplers (e.g., star couplers) [21]. The MINs considered consist of n stages of 2×2 switching elements connecting $N = 2^n$ inputs and outputs. Each input (output) of the MINs is assigned a unique label from $\{0,1\}^n$. The MINs have the property of *full access* that any output is reachable from any input in a single pass through the network. In addition, there is a unique path between any pair of input and output in the MINs. The class of MINs includes several important networks such as butterfly networks, Omega networks, and regular SW bayan networks with spread and fanout of 2 ($S + F = 2$). The parallel computing system considered consists of 2^n processors connected by the MINs as follows: Each processor has an input-port (which receives messages) and an output-port (which sends messages). Each processor is

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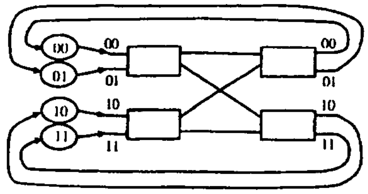

Figure 1: The parallel computing system connected by MINs.

given a unique label of $\{0,1\}^n$ and the input-port (resp. output-port) of the processor is connected to the output (resp. input) with the same label of the MINs. Figure 1 gives an example of the computing system of 2^2 processors. The MINs considered are all-optical. The electro-optic conversion and the storage for messages are available in each processor.

Given an MIN, a routing request $R = \{(u,v)\}$ on the MIN is a partial permutation (resp. permutation) if each input of the MIN appears in R at most once (resp. exactly once) and each output of the MIN appears in R at most once (resp. exactly once). The permutation routing on the MINs with circuit switch routing mode has been extensively studied in the literature. The following results have been known for the permutation routings on the n-dimensional MINs: Finding the optimal number of wavelengths for realizing an arbitrary partial permutation is NP-complete [19]. Any partial permutation can be realized by $2^{\lceil (n-1)/2 \rceil}$ wavelengths and there are permutations which need at least $2^{\lceil (n-1)/2 \rceil}$ wavelengths [1]. Obviously, $2^{\lceil (n-1)/2 \rceil}$ could be far beyond the bandwidth of the MINs. A straightforward approach to realize those permutations is as follows: (1) assign $2^{\lceil (n-1)/2 \rceil}$ virtual wavelengths to the routing paths for R so that the paths which receive the same wavelength are edge-disjoint; (2) partition R into $r = \lceil 2^{\lceil (n-1)/2 \rceil}/W \rceil$ subsets $R_1, ..., R_r$ so that the routing paths for each subset R_i $(1 \le i \le r)$ are assigned at most W wavelengths; and (3) realize each R_i by W wavelengths in one round of routing on the network. This approach, however, may not be efficient. For those permutations, minimizing the number of rounds of routing becomes a key for improving the communication efficiency. A basic idea for this purpose is that in each round of routing, the message for every input-output pair is routed to an intermediate destination and the intermediate destination in the final round of routing is the output of the pair. For the butterfly network, it is known that a permutation can be realized in three rounds of routing by edge-disjoint paths [18]. However, it has been a long standing open problem that if any permutation can be realized in two rounds of routing by edge-disjoint paths on the butterfly network.[1] Several random algorithms for realizing permutation routings in two rounds of routing on the butterfly network have been proposed [25, 24, 4]. If the intermediate destinations are randomly selected, any permutation can be routed with high probability in two rounds of routing on the MINs, each round of routing uses $O(n/\log n)$ wavelengths, where and the throughout this paper, the base of the logarithm is 2 [25, 24].

In this paper, we follow the basic idea to realize permutations in two rounds of routing on the n-dimensional butterfly networks B_n. In contrast to the previous works, we select the intermediate destinations deterministicly. The results hold on the other topologically equivalent networks as well. Our results are:

1. A deterministic algorithm which realizes any partial permutation routing on B_n in at most $\lceil (n-1)/(2\lfloor \log W \rfloor) \rceil$ rounds of routing.
2. A deterministic algorithm which realizes any BPC (bit-permute-complement) permutation routing on B_n in at most two rounds of routing.

Currently, the achievable bandwidth W in practical networks can be as large as 16 [3]. For $W = 16$ and $n \le 17$, $\lceil (n-1)/(2\log W) \rceil \le 2$. Notice that the largest parallel computing system built so far consists of 2^{16} processors [23]. So, our first algorithm realizes any permutation in two rounds of routing on the parallel computing systems connected by MINs with practical size. Although the previous randomized algorithms (e.g., [25, 24, 13, 4]) can realize with high probability any permutation in two rounds of routing on all-optical MINs with practical size, our algorithm is a deterministic one.

The class of BPC permutations is an important class of permutations in parallel processing. It includes perfect shuffle, unshuffle, bit-reversal, butterfly permutations in FFT algorithms, and segment shuffles [12]. Our second algorithms realizes any BPC permutations in two rounds of routing on B_n, each round uses only one wavelength.

In the next section, we give the preliminaries. The algorithms for arbitrary permutations and BPC permutations are given in Sections 3 and 4. The final section concludes the paper.

2 Preliminaries

The n-dimensional butterfly network B_n has n stages and $N = 2^n$ inputs and outputs. Each stage of the butterfly has $N/2$ 2×2 switches (nodes). We number the stages $0, 1, ..., n-1$ from left to right. We label the nodes at stage i $(0 \le i \le n-1)$ by $\langle w, i \rangle$, where w is an $(n-$

[1]It was reported that any permutation can be realized in two rounds of routing by edge-disjoint paths. However, none of those reports is correct and the problem remains open.

1)-bit binary number of $\{0,1\}^{n-1}$ that denotes the *row* of the node. Each node $\langle w,i\rangle$ has two inputs and two outputs. The N inputs (resp. outputs) of the nodes at each stage are labeled by n-bit binary numbers of $\{0,1\}^n$. The inputs of $\langle w,0\rangle$ are the inputs of B_n and the outputs of $\langle w,n-1\rangle$ are the outputs of B_n. There is an edge from node $\langle w,i\rangle$ to node $\langle w',i+1\rangle$ iff either w and w' are identical or w and w' differ in precisely the $(i+1)$th bit from left (the $(i+1)$th most significant bit). More precisely, for node $\langle w,i\rangle$ with $w = x_1...x_{n-1}$ ($1 \le i \le n-2$), if $x_i \oplus x_n = 0$, where $\oplus$ is the exclusive OR of two binary bits, then there is an edge from the output $x_1...x_{n-1}x_n$ of node $\langle w,i-1\rangle$ to the input $x_1...x_{n-1}x_n$ of node $\langle w,i+1\rangle$; and if $x_i \oplus x_n = 1$ then there is an edge from the output $x_1...x_{i-1}x_ix_{i+1}...x_{n-1}x_n$ of node $\langle w,i-1\rangle$ to the input $x_1...x_{i-1}\bar{x}_ix_{i+1}...x_{n-1}\bar{x}_n$ of node $\langle w',i+1\rangle$ where $w' = x_1...x_{i-1}\bar{x}_ix_{i+1}...x_{n-1}$ and $\bar{x}_i$ is the negation of x_i. The 3-dimensional butterfly network B_3 is given in (a) of Figure 2. We call an edge from node $\langle w,i\rangle$ to node $\langle w',i+1\rangle$ a *straight edge* if w and w' are identical, and a *cross edge* otherwise.

Notice that for message routing, the butterfly network is viewed as a directed graph, i.e., the messages can be routed only in the direction from the input to the output. A path in a graph is a sequence of edges of the form $(s_1,s_2)(s_2,s_3)\ldots(s_{k-1},s_k)$, s_i ($1 \le i \le k$) are the nodes of the graph and $s_i \ne s_j$ for $i \ne j$. We sometimes denote the path from s_1 to s_k by $s_1 \to s_k$. Two paths are *edge-disjoint* if they have no common edge.

Given an arbitrary pair (u,v) of input u with binary label $x_1...x_n$ and output v with binary label $y_1...y_n$ in the butterfly, there is a unique path $e_1...e_{n-1}$ from u to v, where e_i is an edge $(\langle w,i-1\rangle,\langle w',i\rangle)$ with $w = y_1...y_{i-1}x_i...x_{n-1}$ and $w' = y_1...y_ix_{i+1}...x_{n-1}$. Given two pairs (u,v) and (u',v') of inputs and outputs, where $u = x_1...x_n$, $v = y_1...y_n$, $u' = x'_1...x'_n$, and $v' = y'_1...y'_n$, the paths $u \to v = e_1...e_{n-1}$ and $u' \to v' = e'_1...e'_{n-1}$ have a common edge if there is an i ($1 \le i \le n-1$) such that $e_i = e'_i$.

Proposition 1 *Given two paths $u \to v = e_1...e_{n-1}$ and $u' \to v' = e'_1...e'_{n-1}$, where $u = x_1...x_n$, $v = y_1...y_n$, $u' = x'_1...x'_n$, and $v' = y'_1...y'_n$, if $e_i = e'_i$ then $y_1...y_{i-1}x_i...x_{n-1}x_n = y'_1...y'_{i-1}x'_i...x'_{n-1}x'_n$ and $y_1...y_ix_{i+1}...x_{n-1}x_n = y'_1...y'_ix'_{i+1}...x'_{n-1}x'_n$.*

Similar to the butterfly B_n defined above, we can define the *backward butterfly*. The differences of the n-dimensional backward butterfly from B_n are: The n stages in the backward butterfly are numbered $n-1, n-2, ..., 1, 0$ from left to right. There is an edge from node $\langle w,i\rangle$ to node $\langle w',i-1\rangle$ iff either w and w' are identical or w and w' differ in precisely the ith bit from right (the ith least significant bit). The 3-dimensional backward butterfly is given in (b) of Figure 2.

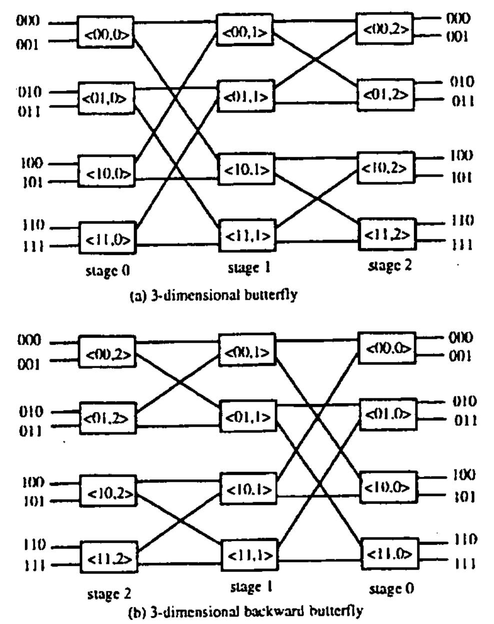

Figure 2: The 3-dimensional butterfly and backward butterfly networks.

The following property of the butterfly is important in this paper. For integers $1 \le j \le (n-1)/k$ and $1 \le k \le n-1$, the subgraph of B_n induced by the nodes at stages from $(n-1)-jk$ to $(n-1)-(j-1)k$ is a collection of $2^{(n-1)-k}$ disjoint $(k+1)$-dimensional butterflies B_{k+1}. The 2-dimensional butterflies induced by the nodes of stages from 0 to 1 and by the nodes of stages from 1 to 2 of B_3 are given in (a) and (b) of Figure 3, respectively. Similarly, the subgraph of the n-dimensional backward butterfly induced by the nodes at stages from $(n-1)-(j-1)k$ to $(n-1)-jk$ is a collection of $2^{(n-1)-k}$ disjoint $(k+1)$-dimensional backward butterflies (see (c) and (d) of Figure 3).

For a permutation R, we often denote the corresponding output of u in R by $R(u)$.

3 Algorithm for arbitrary permutations

The algorithm for arbitrary permutations on the B_n is obtained by implementing the routing paths of the Beneš network for the permutation by multiple rounds of routing on the B_n. The n-dimensional Beneš network BE_n consists of a butterfly B_n and an n-dimensional backward butterfly with the outputs of the B_n connected to the inputs of the backward butterfly as shown in (a) of Figure 4. We call the part of B_n and the backward butterfly in the Beneš network the *forward part* and the *backward part*, respectively. On the other hand, k rounds of routing on B_n can be interpreted as a routing on the k-

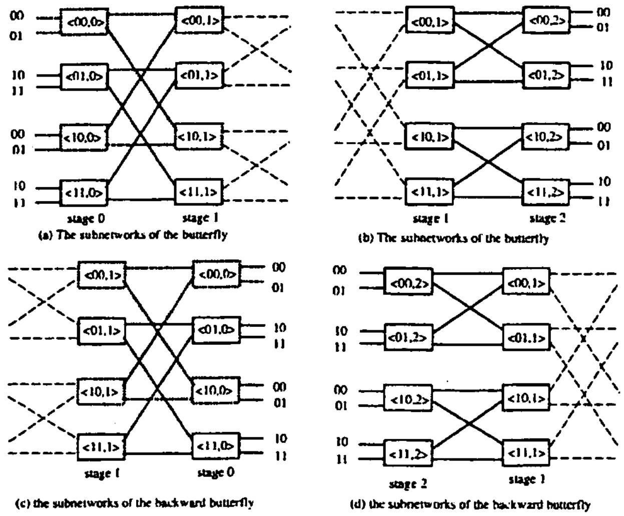

Figure 3: The 2-dimensional subnetworks induced from the 3-dimensional butterfly and backward butterfly.

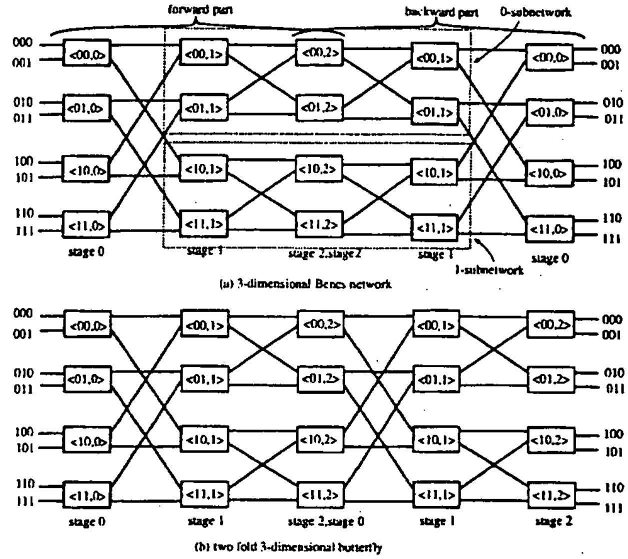

Figure 4: The 3-dimensional Beneš network and the two-fold B_3.

fold B_n ((b) of Figure 4 gives a two-fold B_n). Although the Beneš network and the two-fold butterfly network are closely related in structure, they are not topologically equivalent. It is well known that any permutation on the Beneš network can be realized by edge-disjoint paths (i.e., one wavelength for all-optical Beneš network) [2, 26]. While whether an arbitrary permutation on the two-fold butterfly can be routed by edge-disjoint paths is a long-standing open problem. The advantage of the Beneš network is that permutation routings can be realized by edge-disjoint paths. However, the hardware cost of the Beneš network is as twice as that of B_n. The idea in our algorithm is to implement the edge-disjoint paths of the Beneš network for permutation routing by multiple rounds of routing on B_n. Given a path in the Beneš network, for the segment of the path in the forward part of the network, there is a path in B_n which is isomorphic to the segment. However, the segment of the path in the backward part may not have such a counterpart. How to implement the segments in the backward part of the Beneš network in B_n is a key of our algorithm.

The outline of our algorithm is as follows. Given a permutation routing request $R = \{(u, R(u))\}$ from $\{0,1\}^n$ to $\{0,1\}^n$, we find a set P of edge-disjoint paths in the Beneš network which realize R. Given an integer k ($1 \le k \le n-1$), for each path $u \to R(u)$ of P and $1 \le j \le (n-1)/k$, let $d_j(u)$ be the output of the node at the stage $(n-1) - jk$ of the backward part of the Beneš network that appears in $u \to R(u)$. Taking $d_1(u)$ as the intermediate destination in the 1st round of routing for the request $(u, R(u))$, we route the message from u to $d_1(u)$ by one round of routing on B_n. To do so, let $d(u)$ be the input of the node at the stage $(n-1) - k$ of the B_n that appears in $u \to R(u)$. We first route the message from u to $d(u)$ by the segment $u \to d(u)$ of the path $u \to R(u)$ in P. Next, we route the message from $d(u)$ to $d_1(u)$ on the $(k+1)$-dimensional subnetwork B_{k+1} of B_n. Since the paths in P are edge-disjoint, for all inputs $d(u)$ and outputs $d_1(u)$ of the same $(k+1)$-dimensional subnetwork B_{k+1}, the routing request $\{(d(u), d_1(u))\}$ is a permutation on the B_{k+1}. Therefore, $\{(d(u), d_1(u))\}$ can be realized in $2^{\lceil k/2 \rceil}$ wavelengths on B_{k+1}. Taking $k = 2\lfloor \log W \rfloor$, $\{(d(u), d_1(u))\}$ can be realized in W wavelengths on B_{k+1}. Similarly, for each $j \ge 2$, taking $d_j(u)$ as the intermediate destinations in the jth round of routing, we route the message from $d_{j-1}(u)$ to $d_j(u)$ by one round of routing on B_n. From this, we can route any permutation on B_n in $\lceil (n-1)/(2\lfloor \log W \rfloor) \rceil$ rounds. The formal description of the algorithm is given in Figure 5

Now, we give the formal proof for the algorithm.

Lemma 2 *[2, 26] Any permutation routing on the Beneš network can be realized by edge-disjoint paths.*

A partial permutation $R = \{(u, v)\}$ on B_n is called a *semi-permutation* [28] if for every pair of inputs (resp. outputs) $u = x_0 \ldots x_{n-1}0$ and $u' = x_0 \ldots x_{n-1}1$, at most one of u and u' appears in R as an input (resp. output). In other words, at most one input of every node at stage 0 (resp. at most one output of every node at stage $n-1$) appears in R as an input (resp. output). Given a permutation R, we can partition R into two semi-permutations

Algorithm Arbitrary_Permutation
Input: A permutation routing request $R = \{(u, R(u)\}$ on the B_n with bandwidth W.
Output: The routing paths in $\lceil (n-1)/(2\lfloor \log W \rfloor)\rceil$ rounds of routing on B_n for R and the assignment of wavelengths to the paths.
begin
Construct a set P of edge-disjoint paths which realize R on the BE_n by Lemma 2.
Let $k = 2\lfloor \log W \rfloor$.
For each path $u \to R(u)$ of P, let $d(u)$ be the input of the node at the stage $(n-1)-k$ in the forward part of the BE_n that appears in $u \to R(u)$.
For each path $u \to R(u)$ of P and $1 \le j \le \lceil (n-1)/k \rceil$, let $d_j(u)$ be the output of the node at stage $\max\{(n-1)-jk, 0\}$ in the backward part of the BE_n that appears in $u \to R(u)$.
Let $R_1 = \{(u, d_1(u))\}$ and $R_j = \{(d_{j-1}(u), d_j(u))\}$ for $2 \le j \le \lceil (n-1)/k \rceil$.
Construct the routing paths for R_1 on B_n as follows: for each pair $(u, d_1(u))$, the path $u \to d_1(u)$ consists of the segment $u \to d(u)$ of the path $u \to R(u)$ in P and the unique path $d(u) \to d_1(u)$ in the $(k+1)$-dimensional butterfly B_{k+1} induced by the nodes of stages from $(n-1)-k$ to $n-1$.
Construct the routing paths for R_j $(2 \le j \le \lceil (n-1)/k \rceil)$ on B_n as follows: for each pair $(d_{j-1}(u), d_j(u))$, the path $d_{j-1}(u) \to d_j(u)$ consists of the straight edges from the input of B_n to the input of the $(k+1)$-dimensional butterfly induced by the nodes at stages from $(n-1)-(j-1)k$ and $(n-1)-k$, the unique path $d_{j-1}(u) \to d_j(u)$ in the B_{k+1}, and the straight edges from the output of the B_{k+1} to the output of the B_n.
For the paths constructed above, assign the wavelengths as shown in Lemma 3.
end.

Algorithm BP_Permutation
Input: A BP permutation $R = \{(u, R(u))\}$ on the B_n defined by the permutation π on $\{1, 2, ..., n\}$.
Output: For every $u \in \{0,1\}^n$, an intermediate destination $d(u)$ such that for $u \ne u'$, $d(u) \ne d(u')$, paths $u \to d(u)$ and $u' \to d(u')$ on B_n are edge-disjoint, and paths $d(u) \to R(u)$ and $d(u') \to R(u')$ on B_n are edge-disjoint.
begin
Find all cycles of π.
For each cycle of $j_1, ..., j_k, j_1$, find the minimum integer j from $j_1, ..., j_k$.
$Z_j = X_j$ and $Z_{j_l} = X_{j_l} \oplus Y_{j_l}$ for $1 \le l \le k$ and $j_l \ne j$,
end.

Figure 5: The algorithms for arbitrary permutations and BP permutations on B_n.

as follows: We construct a bipartite graph between the set W_0 of nodes at stage 0 and the set W_{n-1} of nodes at stage $n-1$. There is an edge between a node $w \in W_0$ and a node $w' \in W_{n-1}$ iff there is a routing request $(u, R(u))$ such that u is an input of w and $R(u)$ is an output of w'. The bipartite graph has node-degree at most two. Therefore, the edges of the bipartite graph can be partitioned into two matchings, each match corresponds to a semi-permutation.

Lemma 3 *Any semi-permutation R can be routed in $2^{\lceil (n-1)/2\rceil -1}$ wavelengths on B_n.*

Outline of Proof: For a path $u \to R(u)$ in the butterfly, define the *prefix segment* of length k of $u \to R(u)$ to be the subpath of $u \to R(u)$ consisting of edges from the node at stage $i-1$ to to the node at stage i for $1 \le i \le k$. Define the *suffix segment* of length k of $u \to R(u)$ to be the subpath of $u \to R(u)$ consisting of edges from the node at stage $i-1$ to to the node at stage i for $(n-1)-k \le i \le n-1$. Let P_k and S_k be the set of prefix segments and the set of suffix segments of length k of all routing paths for R, respectively. The lemma is proved by showing that for $1 \le k \le \lceil (n-1)/2 \rceil$, the routing paths for R can be colored by at most 2^{k-1} colors such that the segments of $P_k \cup S_k$ from different paths with the same color are edge-disjoint. □

Notice that Lemma 3 in fact gives an algorithm which assigns 2^{k-1} wavelengths to the paths for realizing a semi-permutation such that the paths received the same wavelength are edge-disjoint. Combining with the procedure for partitioning a permutation into two semi-permutation, we have an algorithm which assigns 2^k wavelengths to the paths for realizing a given permutation such that the paths received the same wavelength are edge-disjoint. This algorithm is used in the algorithm given in Figure 5.

Given a permutation routing request $R = \{(u, R(u))\}$, let P be a set of disjoint paths in the n-dimensional Beneš network that realize R. For each path $u \to R(u)$ of P, $1 \le k \le n-1$, and $1 \le j \le \lceil (n-1)/k \rceil$, let $d_j(u)$ be the output of the node at the stage $\max\{(n-1)-jk, 0\}$ in the backward part of the Beneš network that appears in $u \to R(u)$.

Lemma 4 *Given a permutation R and a set P of edge-disjoint paths which realizes R on the n-dimensional Beneš network, each of the routing requests $R_1 = \{(u, d_1(u))\}$ and $R_j = \{(d_{j-1}(u), d_j(u))\}$ $(2 \le j \le \lceil (n-1)/k \rceil)$ can be realized by $2^{\lceil k/2 \rceil}$ wavelengths on B_n.*

Proof: We first show the lemma for R_1. For each path $u \to R(u)$ of P, let $d(u)$ be the input of the node at the stage $(n-1)-k$ in the forward part of the Beneš network

that appears in $u \rightarrow R(u)$. The path for $(u, d_1(u)) \in R_1$ consists of two segments. The first one is the segment $u \rightarrow d(u)$ of the path $u \rightarrow R(u)$ in P. Since the paths in P are edge-disjoint, the messages from u to $d(u)$ can be routed in one wavelength by the above segments on B_n. The second segment is constructed from the nodes of stage $(n-1)-k$ to the nodes of stage $n-1$ in B_n which routes the message from $d(u)$ to $d_1(u)$. The subgraph of the B_n induced by the nodes from stage $(n-1)-k$ to stage $(n-1)$ consists of $2^{(n-1)-k}$ disjoint $(k+1)$-dimensional B_{k+1}. On the other hand, the subgraph of the n-dimensional Beneš network induced by the nodes between $d(u)$ and $d_1(u)$ consists of $2^{(n-1)-k}$ disjoint $(k+1)$-dimensional subnetworks of the Beneš network, each subnetwork consists of a $(k+1)$-dimensional butterfly B_{k+1} and a $(k+1)$-dimensional backward butterfly. From this, for any path $u \rightarrow R(u)$ in P, $d(u)$ and $d_1(u)$ are in the same subnetwork. Since the paths in P are edge-disjoint, for all inputs $d(u)$ and outputs $d_1(u)$ of the same $(k+1)$-dimensional B_{k+1}, the routing request $\{(d(u), d_1(u))\}$ is a permutation on the B_{k+1}. Therefore, $\{(d(u), d_1(u))\}$ can be realized in $2^{\lceil k/2 \rceil}$ wavelengths on B_{k+1}.

Now we prove the lemma for $R_j = \{(d_{j-1}(u), d_j(u))\}$. The subgraph of the Beneš network induced by the nodes between $d_{j-1}(u)$ and $d_j(u)$ consists of $2^{(n-1)-k}$ disjoint $(k+1)$-dimensional backward butterflies. Each $(k+1)$-dimensional backward butterfly in the Beneš network corresponds to a unique $(k+1)$-dimensional butterfly B_{k+1} induced by the nodes from stage $(n-1)-jk$ to stage $(n-1)-(j-1)k$ of B_n. Considering R_j as the routing request from the inputs of the nodes at stage $(n-1)-jk$ to the outputs of the nodes at stage $(n-1)-(j-1)k$ of the B_n, since the paths in P are edge-disjoint, R_j can be partitioned into $2^{(n-1)-k}$ disjoint permutation routings on the $2^{(n-1)-k}$ $(k+1)$-dimensional butterflies B_{k+1}, one permutation routing per butterfly. The permutation on the B_{k+1} can be realized by $2^{\lceil k/2 \rceil}$ wavelengths. However, we also need to route the messages from the inputs of B_n to the inputs of the B_{k+1} and from the outputs of B_{k+1} to the outputs of B_n by straight edges, each straight edge is used by two paths. To solve the conflicts on the straight edges within the bound of $2^{\lceil k/2 \rceil}$, more work is needed. We use semi-permutations for this purpose. The permutation on each B_{k+1} can be partitioned to two semi-permutations on the B_{k+1}. For each semi-permutation, we first route the message from the inputs $d_{j-1}(u)$ of B_n to the inputs of the B_{k+1} by the straight edges. This can be done by one wavelength due to the property of the semi-permutation. Next we route the message to $d_j(u)$ on the B_{k+1} by $2^{\lceil k/2 \rceil - 1}$ wavelengths (Lemma 3). Finally from the outputs $d_j(u)$ of the B_{k+1} to the outputs $d_j(u)$ of B_n by straight edges. Therefore, each semi-permutation can be realized on B_n by $2^{\lceil k/2 \rceil - 1}$ wavelengths. From this, R_j can be realized on B_n by $2^{\lceil k/2 \rceil}$ wavelengths. □

Theorem 5 *Any permutation routing request R on B_n with optical bandwidth W can be realized in at most $\lceil (n-1)/(2\lfloor \log W \rfloor) \rceil$ rounds of routing.*

Proof: Taking $k = \lfloor 2 \log W \rfloor$, from Lemma 4, each of the routing requests R_j $(1 \le j \le \lceil (n-1)/k \rceil)$ defined in Lemma 4 can be realized in W wavelengths on the B_n. Thus, the theorem holds. □

4 Algorithm for BPC permutations

It has been known that there are BPC permutations which need $2^{\lceil (n-1)/2 \rceil}$ wavelengths on B_n. In this section, we give an algorithm which realizes any BPC permutation by two rounds of routing on B_n, each round of routing uses one wavelength.

A permutation $R = \{(u, R(u))\}$ on $\{0,1\}^n$ is called a BPC permutation if for any input $u = x_1 x_2 \ldots x_n$,

$$R(x_1 x_2 \ldots x_n) = y_1 y_2 \ldots y_n,$$

where $y_j = x_{i_j}$ or $y_j = \bar{x}_{i_j}$ $(1 \le j \le n)$ and $\{i_1, i_2, \ldots, i_n\}$ is a permutation of $\{1, 2, \ldots, n\}$. R is called a BP permutation if $y_j = x_{i_j}$ $(1 \le j \le n)$ (i.e., $R(x_1 x_2 \ldots x_n) = x_{i_1} x_{i_2} \ldots x_{i_n}$).

Recall that for a pair (u, v) of input and output in B_n with $u = x_1 \ldots x_n$ and $v = y_1 \ldots y_n$, there is a unique path $u \rightarrow v = e_1 \ldots e_{n-1}$ where e_i is an edge $(\langle w, i-1 \rangle \langle w', i \rangle)$ with $w = y_1 \ldots y_{i-1} x_i \ldots x_{n-1}$ and $w' = y_1 \ldots y_i x_{i+1} \ldots x_{n-1}$. For any two pairs (u, v) and (u', v') of inputs and outputs in B_n with $u = x_1 \ldots x_n$, $v = y_1 \ldots y_n$, $u' = x'_1 \ldots x'_n$, and $v' = y'_1 \ldots y'_n$, the paths $u \rightarrow v = e_1 \ldots e_{n-1}$ and $u' \rightarrow v' = e'_1 \ldots e'_{n-1}$ are edge-disjoint if for all $1 \le i \le n-1$, $e_i \ne e'_i$.

Lemma 6 *Let R be the BPC permutation and R' be the BP permutation on B_n defined by the same permutation $\{i_1, i_2, \ldots, i_n\}$ of $\{1, 2, \ldots, n\}$. For any $u = x_1 \ldots x_n, u' = x'_1 \ldots x'_n \in \{0,1\}^n$, paths $u \rightarrow R(u)$ and $u' \rightarrow R(u')$ in B_n are edge-disjoint iff paths $u \rightarrow R'(u)$ and $u' \rightarrow R'(u')$ in B_n are edge-disjoint.*

Proof: Omitted. □

From the above lemma, to realize a BPC permutation on the all-optical butterfly network, it is sufficient to consider the routing of the corresponding BP permutation on the network. In the rest of this paper, we consider only BP permutations. We show that for any BP permutation $R = \{(u, R(u))\}$ on the B_n, we can find a set of 2^n intermediate destinations such that the messages from u to the intermediate destinations can be routed by edge-disjoint paths on B_n and the messages from the intermediate destinations to $R(u)$ can be routed by edge-disjoint paths on B_n as well. This implies that R can be

routed in two rounds of routing on B_n for any n, each round of routing uses one wavelength.

Let $R = \{(u, R(u))\}$ be a BP permutation on B_n such that for $u = x_1...x_n$, $R(u) = y_1...y_n$. To get our result, we need to find, for every $u \in \{0,1\}^n$, the intermediate destination $d(u) = z_1...z_n$ such that for any u and u' with $u \neq u'$, $d(u) \neq d(u')$, paths $u \to d(u)$ and $u' \to d(u')$ are edge-disjoint, and paths $d(u) \to R(u)$ and $d(u') \to R(u')$ are edge-disjoint.

Considering each binary label $x_1...x_n$ as a row of n binary bits. The 2^n binary labels form an $2^n \times n$ array. For $1 \leq i \leq n$, we use X_i to denote the ith column of the array. Similarly, Y_i (resp. Z_i) denotes the ith column of the array consists of the 2^n rows of labels $y_1...y_n$ (resp. $z_1...z_n$). For a BP permutation R defined by the permutation $\pi(12...n) = (i_1 i_2...i_n)$, the array $Y_1Y_2...Y_n = X_{i_1}X_{i_2}...X_{i_n}$. To find the intermediate destination $d(u)$ for every u, we construct the array $Z_1Z_2...Z_n$ such that the jth row of $Z_1Z_2...Z_n$ is the intermediate destination of the jth row of $X_1X_2...X_n$.

For $u = x_1...x_n$, $u' = x'_1...x'_n$, $d(u) = z_1...z_n$, and $d(u') = z'_1...z'_n$, let $e_1...e_{n-1}$ be the edges of path $u \to d(u)$ and $e'_1...e'_{n-1}$ be the edges of path $u' \to d(u')$. Then edge e_i $(1 \leq i \leq n-1)$ has the form

$$(\langle z_1...z_{i-1}x_i...x_{n-1}, i-1\rangle, \langle z_1...z_i x_{i+1}...x_{n-1}, i\rangle)$$

and edge e'_i has the form

$$(\langle z'_1...z'_{i-1}x'_i...x'_{n-1}, i-1\rangle, \langle z'_1...z'_i x'_{i+1}...x'_{n-1}, i\rangle).$$

From Proposition 1, $e_i = e'_i$ implies $z_1...z_i x_{i+1}...x_n = z'_1...z'_i x'_{i+1}...x'_n$. Similarly, for $R(u) = y_1...y_n$ and $R(u') = y'_1...y'_n$, let $e_1...e_{n-1}$ be the edges of path $d(u) \to R(u)$ and $e'_1...e'_{n-1}$ be the edges of path $d(u') \to R(u')$. Then $e_i = e'_i$ implies $y_1...y_i z_{i+1}...z_n = y'_1...y'_i z'_{i+1}...z'_n$. We construct the array $Z_1Z_2...Z_n$ in such a way that for any i $(1 \leq i \leq n)$ $z_1...z_i x_{i+1}...x_n = z'_1...z'_i x'_{i+1}...x'_n$ iff $x_1...x_n = x'_1...x'_n$ (and $y_1...y_i z_{i+1}...z_n = y'_1...y'_i z'_{i+1}...z'_n$ iff $y_1...y_n = y'_1...y'_n$). To construct the array $Z_1Z_2...Z_n$ as above, the *cycles* of the permutation π are used. Given a permutation π on $\{1, 2, ..., n\}$, the sequence $j_1, j_2, ..., j_k, j_1$ is called a cycle of π if $\pi(j_l) = j_{l+1}$ for $1 \leq l \leq k-1$ and $\pi(j_k) = j_1$. Any permutation π on $\{1, 2, ..., n\}$ can be partitioned into disjoint cycles. For each cycle $j_1, j_2, ..., j_k, j_1$, $X_{j_l} = Y_{j_{l+1}}$ for $1 \leq l \leq k-1$ and $X_{j_k} = Y_{j_1}$. Assume that we take $Z_{j_l} = X_{j_l} \oplus Y_{j_l}$, where $\oplus$ is the bitwise exclusive OR of the two columns X_{j_l} and Y_{j_l}. Then for any nonempty $J \subseteq \{j_1, ..., j_k\}$, $x_j = x'_j$ $(j \in J)$ and $z_i = z'_i$ $(i \in \{j_1...j_k\} \setminus J)$ implies $x_l = x'_l$ $(l \in \{j_1...j_k\})$. Similar result holds for y_l and y'_l $(l \in \{j_1...j_k\})$. Notice that if we take $Z_{j_l} = X_{j_l} \oplus Y_{j_l}$ for all $l \in \{j_1...j_k\}$ then for $i \geq \max\{j_1, ..., j_k\}$, we do not have a nonempty $J \subseteq \{j_1, ..., j_k\}$ with $x_j = x'_j$ $(j \in J)$. To get at least one $x_j = x'_j$ when $z_{j_l} = z'_{j_l}$ for all $l \in \{j_1, ..., j_k\}$, we take $Z_j = X_j$ for the minimum integer j of $\{j_1...j_k\}$. The algorithm for constructing the array $Z_1Z_2...Z_n$ is given in Figure 5.

Theorem 7 *For any BP permutation R defined by the permutation π on $\{1, 2, ..., n\}$, let $Z_1...Z_n$ be constructed in Algorithm BP_permutation. Then for any $u = x_1...x_n$ and $u' = x'_1...x'_n$ with $u \neq u'$, $d(u) = z_1...z_n \neq d(u') = z'_1...z'_n$, paths $u \to d(u)$ and $u' \to d(u')$ on B_n are edge-disjoint, and paths $d(u) \to R(u)$ and $d(u') \to R(u')$ on B_n are edge-disjoint.*

Outline of the Proof: We first show the statement that for any $1 \leq i \leq n$,

$$z_1...z_i x_{i+1}...x_n = z'_1...z'_i x'_{i+1}...x'_n \text{ iff } x_1...x_n = x'_1...x'_n.$$

This statement implies that for any $u = x_1...x_n$ and $u' = x'_1...x'_n$ with $u \neq u'$, $d(u) = z_1...z_n \neq d(u') = z'_1...z'_n$ and paths $u \to d(u)$ and $u' \to d(u')$ on B_n are edge-disjoint. We prove this statement by induction on i. For $i = 1$, since $Z_1 = X_1$, $z_1x_2...x_n = z'_1x'_2...x_n$ iff $x_1...x_n = x'_1...x'_n$.

Assume that the statement is true for $i - 1 \geq 1$ and we prove it for i. If $Z_i = X_i$ then $z_i = x_i$ and $z'_i = x'_i$. From this and the induction hypothesis, $x_1...x_n = x'_1...x'_n$. Assume that $Z_i = X_i \oplus Y_i$. Let the sequence $j_1, ..., j_k, j_1$ be the cycle of π that $i \in \{j_1, ..., j_k\}$. Without loss of generality, we assume that j_1 is the minimum integer of the cycle. Then from the construction of $Z_1...Z_n$, $z_{j_1} = x_{j_1}$, $z'_{j_1} = x'_{j_1}$, and for $2 \leq l \leq k$, $z_{j_l} = x_{j_l} \oplus y_{j_l}$ and $z'_{j_l} = x'_{j_l} \oplus y'_{j_l}$. From $z_1...z_i x_{i+1}...x_n = z'_1...z'_i x'_{i+1}...x'_n$, we have $x_{j_1} = x'_{j_1}$. For $1 \leq l \leq k-1$, from $\pi(j_l) = j_{l+1}$, we have $x_{j_l} = y_{j_{l+1}}$. From this, $x_{j_1} = x'_{j_1}$, and $z_1...z_i x_{i+1}...x_n = z'_1...z'_i x'_{i+1}...x'_n$, we get $x_{j_l} = x'_{j_l}$ for $2 \leq l \leq k$. Since $i \in \{j_1, ..., j_k\}$, $x_i = x'_i$. Therefore, from the induction hypothesis, $x_1...x_n = x'_1...x'_n$. Obviously, if $x_1...x_n = x'_1...x'_n$ then $z_1...z_i x_{i+1}...x_n = z'_1...z'_i x'_{i+1}...x'_n$.

Following a similar argument, we can show that if $u \neq u'$ then paths $d(u) \to R(u)$ and $d(u') \to R(u')$ on B_n are edge-disjoint. □

5 Concluding remarks

Previous studies showed that the number of wavelengths for some permutation routings on the MINs may be far beyond the optical bandwidth of the network and multiple rounds of routing are needed to realize such permutations. This gives an important optimization problem on minimizing the number of rounds of routing to realize such permutations. In this paper, we gave an algorithm which realizes any permutation on the n-dimensional butterfly networks in $\lceil (n-1)/(2\lfloor \log W \rfloor) \rceil$ rounds of routing. For the parallel computing systems with practical size, the algorithm realizes any permutation in at most two rounds of routing. We also gave an algorithm which realizes any BPC permutation in two rounds of routing, each

round of routing uses one wavelength. An interesting open problem is that if any permutation can be realized in two rounds of routing on the butterfly networks, each round of routing uses $O(1)$ wavelengths.

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